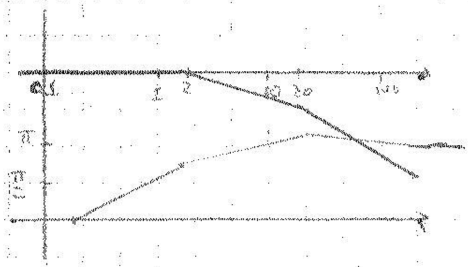
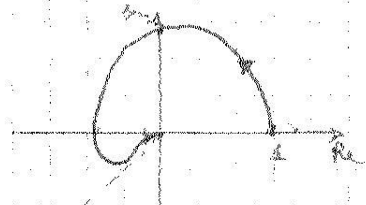
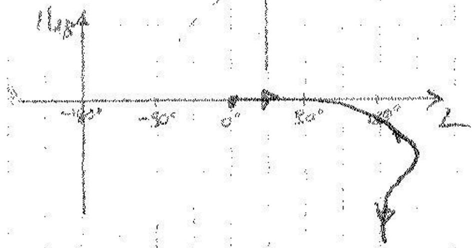
FONDAMENTI DI AUTOMATICA 16/7/2009

1) $G(s)=\dfrac{1+\frac{s}{2}}{\left(1+\frac{s}{20}\right)\left(1-\frac{s}{2}\right)^2}$







NON AS. STABILE ⇒ NON È UN FILTRO

2) a) $S1:\begin{cases}\begin{pmatrix}\dot x_1\\ \dot x_2\end{pmatrix}=\begin{pmatrix}0 & 1\\ -2 & -2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}+\begin{pmatrix}0\\ 1\end{pmatrix}u\\ y_1=\begin{pmatrix}0 & 2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}\end{cases}$ $\quad S2:\begin{cases}\dot x_3=-\frac{1}{4}x_3+u_2\\ y=2x_3\end{cases}$ $\quad u_2=y_1$

$\Rightarrow \begin{pmatrix}\dot x_1\\ \dot x_2\\ \dot x_3\end{pmatrix}=\begin{pmatrix}0 & 1 & 0\\ -2 & -2 & 0\\ 0 & 2 & -\frac{1}{4}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}+\begin{pmatrix}0\\ 1\\ 0\end{pmatrix}u$

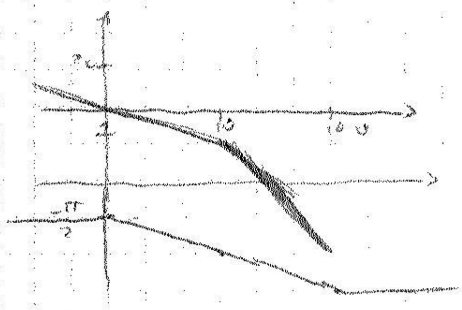
$y=\begin{pmatrix}0 & 0 & 2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$

b) $G(s)=\dfrac{16s}{(1+4s)(s^2+2s+2)}$ $\quad y(t)=G(0)\cdot 2+2\cdot|G(j1)|\sin(t+0.3+\angle G(j1))$

$G(0)=\emptyset \qquad |G(j1)|=1,43 \qquad \angle G(j1)=-0.86$

c) $t>5 \qquad x_3(5)=\dfrac{y(5)}{2}=-1,67 \qquad y(t)=2\cdot x_3(5)\,e^{-\frac{1}{4}(t-5)}\,1(t-5)$

3) $F(s)=\dfrac{100}{s(s+10)^2}$



$m_\varphi \cong 80°$

$m_\alpha = 26\,dB \quad (20)$

$r(t)=4\cdot 1(t) \to e_y(\infty)=0$

$r(t)=4t\cdot 1(t) \to e_y(\infty)=4$

$d(t)=4\cdot 1(t) \to e_y(\infty)=-0.4$

4) $\begin{cases}\bar x_1^3+\bar x_1=0\\ \bar x_2=\bar x_3\\ \bar x_3=-\bar x_1+\bar x_2\end{cases} \Rightarrow \begin{cases}\bar x_1(\bar x_1^2+1)=0\\ \bar x_2=\bar x_3\\ \bar x_3=\bar x_2-\bar x_1\end{cases} \Rightarrow \begin{cases}\bar x_1=0\\ \bar x_2=\bar x_3\\ \bar x_3=\bar x_2\end{cases} \Rightarrow \bar X=\begin{pmatrix}0\\ a\\ a\end{pmatrix}$

con $a\in R$

Qualsiasi sia lo stato di equilibrio $\Rightarrow \begin{pmatrix}x_1(k+1)\\ x_2(k+1)\\ x_3(k+1)\end{pmatrix}=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ -1 & 1 & 0\end{pmatrix}\begin{pmatrix}x_1(k)\\ x_2(k)\\ x_3(k)\end{pmatrix}+\begin{pmatrix}1\\ 0\\ 0\end{pmatrix}u(k)$

$y(k)=\begin{pmatrix}0 & \cos(a) & -\cos(a)\end{pmatrix}\begin{pmatrix}x_1(k)\\ x_2(k)\\ x_3(k)\end{pmatrix}$

SEMPLICEMENTE STABILE (autovalori: 0, 1, -1)